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Theory of Light Scattering from Coupled Electromagnetic-Magnetoplasma Modes in Semiconductors

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In a magnetic field, plasma waves are partially transverse and can couple to electromagnetic waves. This interaction is investigated, and the possibility of observing such mixed modes in doped semiconductors, via light scattering, is considered. For small wave vector, the mode coupling is strong, giving rise to a mixed excitation which we term a plasma-polariton. Unfortunately, the scattering strength for this entity is small and it will be difficult to observe.

I. INTRODUCTION

THE scattering of light from plasmas in semiconductors has been intensively studied during the past few years. This problem was first attacked theoretically in the papers of Platzman[1] and McWhorter.[2] Subsequently, Mooradian and Wright[3] used light scattering to directly observe the coupled plasma-phonon system in n-type samples of GaAs. Their experiment was the first of several in this area. Plasmon light scattering has since been observed in a considerable variety of semiconductors.[4] It has been used to study the dispersion and Landau damping[5] of these waves, and detailed measurements of intensity and polarization[6] have been correlated with theory.

More recently, the question of light scattering from magnetoplasma waves has begun to be investigated. Platzman, Tzoar, and Wolff[7] have pointed out that in a magnetoplasma, light scattering can be used to study the coupling between the hybrid mode and Bernstein[8] modes of the electron gas. These coupling effects have since been demonstrated experimentally by Patel and Slusher.[9] Their work is the first direct verification of the existence of Bernstein modes in solid-state plasmas. It is important to realize that the coupling which makes possible the observation of the Bernstein modes is a finite wave-vector effect. The modes are uncoupled when their wavelength is large.

In this paper we wish to consider the problem of light scattering from magnetoplasma in the opposite limit, namely that of very small wave vector. In this limit there is no coupling to the Bernstein modes, but the plasma waves can interact with electromagnetic modes, in a manner very similar to that in which the phonon couples to the electromagnetic field to generate a polariton.[10] We will call the excitation which results from the interaction of the electromagnetic and plasma waves a "plasma-polariton." The study of the dispersion relation and scattering strengths for this entity is the main purpose of this paper.

To this end, we wish to develop a general formula describing the scattering of light from magnetoplasma waves, particularly waves whose velocities are sufficiently large that they can interact with electromagnetic waves. The derivation, though it parallels one given by McWhorter,[2] will be presented in detail because our approach differs somewhat from his. In the case of a single component magnetoplasma, the result can be considerably simplified compared to his. The formula also simplifies in the limit where the plasma waves travel slowly compared to the velocity of light. Here we will make contact with the work of Platzman, Tzoar, and Wolff,[7] who used the low phase velocity limit of the scattering formula. This limit is often called the quasi-static limit.

In Sec. III, we will use the light scattering formula to discuss the plasma-polariton dispersion relations, and will obtain expressions for the scattering strengths of these modes. The results indicate that, for small q, there is strong coupling between the magnetoplasma and electromagnetic waves. One example is worked out in detail. Plasma-polaritons will not be easy to study, however. The coupling which gives rise to the mixed modes is only strong for small wave vectors, which means the scattering experiment must be done near the forward direction. Unfortunately, forward scattering from plasmas waves is very weak. This fact will make the observation of this mode quite difficult.

Finally, in the last section of the paper, we discuss the oscillator strengths for hybrid modes which propagate at 45° to a magnetic field in the magnetoplasma. These modes have recently been observed in GaAs by Patel and Slusher.[9]

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[7] P. M. Platzman, N. Tzoar, and P. A. Wolff, Phys. Rev. **174**, 489 (1968).
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[10] O. Fano, Phys. Rev. **103**, 1202 (1956); J. J. Hopfield, *ibid.* **112**, 1555 (1958); C. H. Henry and J. J. Hopfield, Phys. Rev. Letters **15**, 964 (1965).

II. LIGHT-SCATTERING FORMULA

In studying the light-scattering problem, we will consider a single-component magnetoplasma whose motion is described by the Hamiltonian

$$H=\sum_i\left(\frac{\mathbf{p}_i-(e/c)\mathbf{A}_i}{2m^*}\right)^2+\tfrac{1}{2}\sum_{i\neq j}\left(\frac{e^2}{\epsilon_0 r_{ij}}\right). \qquad (1)$$

It is well known that the spectrum of radiation scattered from such a plasma is determined by the Fourier transform of the electron density-density correlation function in the medium. Specifically, the spectrum is given by

$$\frac{d^2\sigma}{d\Omega d\omega}=\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{\omega_1}{\omega_0}\right)\int_{-\infty}^{\infty}\langle\rho(\mathbf{q},t)\rho(-\mathbf{q},0)\rangle e^{i\omega t}\frac{dt}{2\pi}, \qquad (2)$$

in which ρ is the time-dependent electron-density operator, $\mathbf{q}$ is the wave vector transferred to the plasma in the scattering, and ω is the frequency transfer. It is important to realize that the spectrum is a function only of the wave vector and frequency transfers. In the solid-state case, Eq. (2), and the Hamiltonian which precedes it, are valid when the frequencies (ω_0 and ω_1) of incident and scattered light waves are low compared to the energy band gaps. However, the same formula is approximately correct at finite frequencies if one replaces the Thomson cross section $(e^2/m^*c^2)^2$ by an enhanced Thomson cross section[11]:

$$\sigma=\left(\frac{e^2}{m^*c^2}\right)^2\left[\frac{E_G{}^2}{E_G{}^2-(\hbar\omega_0)^2}\right]^2. \qquad (3)$$

In the following, our principal problem will be the calculation of the density-density correlation function which is central to Eq. (2). It is usually most convenient to determine the closely related response function G. In thermal equilibrium G is related to the correlation function through the fluctuation dissipation theorem:[12]

$$\int_{-\infty}^{\infty}\langle\rho(\mathbf{q},t)\rho(-\mathbf{q},0)\rangle\frac{e^{i\omega t}dt}{2\pi}\equiv J(\omega)=\left[\frac{-2\,\mathrm{Im}[G(\omega)]}{1-e^{-\beta\omega}}\right]. \qquad (4)$$

G is easier to calculate than J. In addition, it has a simple and helpful physical interpretation. G determines the electron density that is induced in a plasma by an external electrostatic potential of wave vector $\mathbf{q}$ and frequency ω. This relation is shown in Eq. (5):

$$\rho_{\mathrm{ind}}(\mathbf{q},\omega)=2\pi eG\varphi_{\mathrm{ext}}(\mathbf{q},\omega). \qquad (5)$$

Equations (4) and (5) are exact, but approximations will be necessary to calculate G. Here our primary tool will be the random phase approximation. In its simplest form, this approximation states that the plasma responds as a noninteracting electron gas to the *total* potential that is present. The key word here is "total." In Eq. (5), the potential which drives the plasma is an *external* one, φ_{ext}. φ_{ext} induces a charge density in the electron gas which, in turn, via Poisson's equation, produces an *induced* potential. The combination, $\varphi_{\mathrm{total}}=\varphi_{\mathrm{ext}}+\varphi_{\mathrm{ind}}$, is the total potential to which the plasma responds. This, at least, is the case in a plasma in which electromagnetic effects are unimportant. In the circumstances we will be considering, the situation is complicated by the fact that induced currents in the plasma generate a vector potential, in addition to the electrostatic potential mentioned above. *Both* of these potentials must be determined self-consistently from the induced charge and current in the plasma, and from Maxwell's equations. It is this problem to which we now turn our attention.

In Eq. (6) we write the total electric field in the plasma as the sum of three terms: the gradient of an external electrostatic potential, the gradient of an induced electrostatic potential, and the time derivative of an induced vector potential:

$$\mathbf{E}_{\mathrm{total}}=-\nabla\varphi_{\mathrm{ext}}-\nabla\varphi_{\mathrm{ind}}-\frac{1}{c}\frac{\partial\mathbf{A}_{\mathrm{ind}}}{\partial t}. \qquad (6)$$

The two induced potentials are determined by the induced charge density and the transverse part of the induced current, via Maxwell's equations:

$$-\nabla^2\varphi_{\mathrm{ind}}=4\pi e\langle\rho_{\mathrm{ind}}\rangle,$$

$$-\nabla^2\mathbf{A}_{\mathrm{ind}}+\frac{1}{c^2}\frac{\partial^2\mathbf{A}_{\mathrm{ind}}}{\partial t^2}=\frac{4\pi\langle\mathbf{j}_{\mathrm{trans}}\rangle}{c}. \qquad (7)$$

The induced current in the plasma is related to the total electric field by the plasma conductivity tensor $\boldsymbol{\sigma}(\mathbf{q},\omega)$;

$$\mathbf{j}_{\mathrm{ind}}(\omega)=\boldsymbol{\sigma}\cdot\mathbf{E}_{\mathrm{total}}. \qquad (8)$$

In calculating $\boldsymbol{\sigma}$, which determines the response of the plasma to the *total* electric field, we will treat the medium as a noninteracting gas. This is the basic approximation of the random phase approximation. We may now use the continuity equation to relate the induced charge density to the divergence of the induced current. It is then possible to combine the two Maxwell equations in the form

$$\left(q^2-\mathbf{q}\mathbf{q}-\frac{\omega^2}{c^2}\right)\mathbf{E}_{\mathrm{ind}}=\frac{4\pi i\omega}{c^2}\mathbf{j}_{\mathrm{ind}}. \qquad (9)$$

With the aid of Eqs. (8) and (9), the induced electric field and induced charge density in the plasma may now be related to the external field. For the induced field we

[11] A. L. McWhorter and P. N. Argyres, Bull. Am. Phys. Soc. **12**, 102 (1967); A. L. McWhorter and P. N. Argyres, Paper D-6, International Conference on Light Scattering, NYU, 1968 (unpublished).

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find

$$\left[q^2-\mathbf{qq}-\frac{\omega^2}{c^2}\left(1-\frac{4\pi\boldsymbol{\sigma}}{i\omega}\right)\right]\mathbf{E}_{\text{ind}}=\frac{4\pi i\omega}{c^2}\boldsymbol{\sigma}\cdot\mathbf{E}_{\text{ext}}. \quad (10)$$

An expression for the induced charge may be obtained by combining Eqs. (8) and (10) and the continuity equation. This is the quantity we really need, since it determines the Green's function via Eq. (5). The final result for G is

$$G=\frac{1}{2\pi e^2}\left\{\left(\frac{4\pi}{c^2}\right)(\mathbf{q}\cdot\boldsymbol{\sigma}\cdot\mathbf{D}^{-1}\cdot\boldsymbol{\sigma}\cdot\mathbf{q})-\left(\frac{i\mathbf{q}\cdot\boldsymbol{\sigma}\cdot\mathbf{q}}{\omega}\right)\right\}, \quad (11)$$

where the tensor $\mathbf{D}$ is defined by $\mathbf{D}=q^2-\mathbf{qq}-\omega^2/c^2+4\pi\omega\boldsymbol{\sigma}/ic^2$. Equation (11) can be considerably simplified since $\boldsymbol{\sigma}$ is directly related to $\mathbf{D}$. After a little algebra, one finds that G is given by the expression

$$G=\left(\frac{1}{2\pi}\right)\left(\frac{i\omega}{c^2}\right)\left[q^2\frac{ic^2}{4\pi\omega}+(\mathbf{q}\cdot\mathbf{D}^{-1}\cdot\mathbf{q})\left(\frac{i\omega}{4\pi}\right)\right]. \quad (12)$$

Lastly, we may take the imaginary part of G to calculate the spectrum of the scattered radiation:

$$\frac{d^2\sigma}{d\Omega d\omega}=\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{1}{4\pi^2e^2}\right)\left(\frac{\omega^2}{c^2}\right)\frac{\text{Im}[\mathbf{q}\cdot\mathbf{D}^{-1}\cdot\mathbf{q}]}{[1-e^{-\beta\omega}]}. \quad (13)$$

This formula is our basic result which will be used throughout the rest of the paper to discuss various sorts of scattering experiments. It is only valid for the case of a single component plasma. A more complicated correlation function is required[2] to describe light scattering from multicomponent plasmas.

III. PLASMA-POLARITON SCATTERING

In Sec. II we have derived a general formula, Eq. (13), which describes the scattering of light from a single component plasma. We now wish to study the predictions of this equation in a number of cases. A case of particular interest is that in which electromagnetic waves and plasma waves are coupled via dc magnetic field applied to the plasma. This coupled excitation is what we earlier termed a plasma-polariton.

However, before considering the plasma-polariton scattering, it is worthwhile to see what our formula predicts in the simple case of zero magnetic field. Under these circumstances the dielectric tensor appearing in Eq. (13) is diagonal, and so is $\mathbf{D}$. The vector product simplifies greatly, as shown below:

$$\mathbf{q}\cdot\mathbf{D}^{-1}\cdot\mathbf{q}=-(q^2c^2/\omega^2)(1/\epsilon), \quad (14)$$

where

$$\epsilon(\mathbf{q},\omega)\equiv[1-4\pi\boldsymbol{\sigma}(\mathbf{q},\omega)/i\omega]$$

is the plasma dielectric function. The differential cross section is

$$\frac{d^2\sigma}{d\Omega d\omega}=\left(\frac{1}{\pi}\right)\left(\frac{q^2}{4\pi e^2}\right)\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{\omega_1}{\omega_0}\right)\frac{\text{Im}(1/\epsilon)}{[1-e^{-\beta\omega}]}. \quad (15)$$

This expression is the well-known formula for the light-scattering cross section of an unmagnetized, single-component plasma. Its predictions have been discussed in detail in various places in the literature.

Now we consider Eq. (13) in the case when a dc magnetic field is present. The dielectric tensor is no longer diagonal, so we must face the problem of inverting the tensor $\mathbf{D}$. For this purpose it is convenient to write the wave vector $\mathbf{q}$ as the product of an effective index of refraction n and a unit vector $\boldsymbol{\alpha}$ as shown below:

$$n^2=c^2q^2/\omega^2, \quad \mathbf{q}=(n\omega/c)\boldsymbol{\alpha}. \quad (16)$$

With these definitions $\mathbf{D}$ can be written in the form

$$\mathbf{D}=\left(\frac{\omega^2}{c^2}\right)[n^2(1-\boldsymbol{\alpha\alpha})-\boldsymbol{\varepsilon}]\equiv\left(\frac{\omega^2}{c^2}\right)\mathbf{F}. \quad (17)$$

The inverse of $\mathbf{D}$ is proportional to the inverse of $\mathbf{F}$ which, in turn, is given by the well-known formula for the inverse of a matrix:

$$\mathbf{F}^{-1}=\frac{\text{cofactor }(\mathbf{F})}{\det\ (\mathbf{F})}. \quad (18)$$

A straightforward expansion shows that sixth-order terms in n drop out of the expression for the determinant of $\mathbf{F}$ and it reduces to the form given below:

$$\det\mathbf{F}=An^4+Bn^2+C,$$

with

$$A=\boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\alpha},$$
$$B=(\boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\alpha})-(\boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\alpha})\ \text{Tr}(\boldsymbol{\varepsilon}), \quad (19)$$
$$C=\det(\boldsymbol{\varepsilon}).$$

The algebraic manipulations which lead to these formulas for the coefficients A, B, and C are straightforward, though tedious in the case of B. Equation (19) has zeros which determine the collective modes of the coupled plasma and electromagnetic field.

With the aid of Eq. (17) it is now possible to write out a detailed expression for the inverse of the tensor $\mathbf{F}$. We will not display this result in its most general form since we have no use for it. However, it is of some interest to consider the contributions of the highest order terms in n (assuming $n\gg1$) to the cofactor. A simple calculation shows that they have the form shown below:

$$\text{cof}(\mathbf{F})\simeq n^4(\alpha_i\cdot\alpha_j). \quad (20)$$

Thus, in the limit where n is very large compared to unity, the tensor product (Eq. 13) takes the simple form

$$(\mathbf{q}\cdot\mathbf{D}^{-1}\cdot\mathbf{q})=(c^2q^2/\omega^2)[1/(\boldsymbol{\alpha}\cdot\boldsymbol{\varepsilon}\cdot\boldsymbol{\alpha})]. \quad (21)$$

This is the formula that was used in the analysis of Platzman, Tzoar, and Wolff.[7] It is valid for large n; that is, when the mode under consideration is traveling slowly compared to the velocity of light. Modes of this sort are normally excited in large angle light scattering experiments. It is only for near forward scattering that the condition $n \gg 1$ is violated, and one must consider the detailed coupling of the magnetoplasma wave to the electromagnetic wave. This is the question to which we now turn our attention.

To study the mode coupling which gives rise to the plasma-polariton, we choose a particularly simple geometry—that in which the wave vector $\mathbf{q}$ is perpendicular to the dc magnetic field. We will also assume that $\mathbf{q}$ is sufficiently small that local expressions for the dielectric tensor may be used in calculating $\mathbf{F}$. Under these conditions $\mathbf{F}$ takes the following form:

$$\mathbf{F}=\begin{bmatrix} -\epsilon_{\perp} & \epsilon_{\times} & 0 \\ -\epsilon_{\times} & n^2-\epsilon_{\perp} & 0 \\ 0 & 0 & n^2-\epsilon_z \end{bmatrix}, \tag{22}$$

where $\epsilon_{\perp}$, $\epsilon_{\times}$, and ϵ_z are the well-known[13] components of the dielectric tensor of a magnetized plasma:

$$\epsilon_{\perp}=\left[1-\frac{\omega_p^2}{\omega^2-\omega_c^2}\right],$$
$$\epsilon_{\times}=\frac{i\omega_c\omega_p^2}{\omega(\omega^2-\omega_c^2)}, \quad \epsilon_z=\left[1-\frac{\omega_p^2}{\omega^2}\right]. \tag{23}$$

Here ω_c and ω_p are the cyclotron and plasma frequencies, respectively. For simplicity, we have assumed that the collision frequency of the carriers is very small. Collisions could be included in a phenomenological way, though the resulting formulas become quite cumbersome. The tensor $\mathbf{F}$ may now be inverted to obtain an expression for the differential cross section. First, however, we consider the frequencies of the coupled modes that are determined by the equation $(\det)(\mathbf{F})\equiv 0$. They turn out to be[13]

$$\omega_{1,2}^2=\tfrac{1}{2}(c^2q^2+\omega_c^2+2\omega_p^2)\pm\tfrac{1}{2}[(c^2q^2-\omega_c^2)^2+4\omega_c^2\omega_p^2]^{1/2}. \tag{24}$$

In the limit of large q, ω_1 and ω_2 take the form

$$\omega_1^2\simeq c^2q^2,$$
$$\omega_2^2\simeq(\omega_c^2+\omega_p^2). \tag{25}$$

Here the electromagnetic and hybrid plasma waves are essentially decoupled from one another. It is only for small q, i.e., for forward scattering, that the frequencies are strongly modified by interaction. Figure 1 shows plots of the frequencies ω_1 and ω_2 as a function of q for the particular case in which the cyclotron frequency is equal to the plasma frequency. Similar curves can be

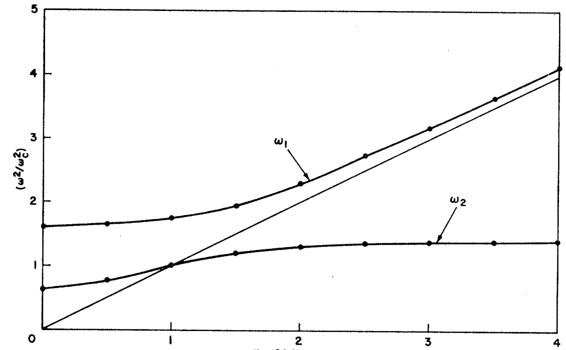

FIG. 1. The plasma-polariton dispersion relation for the case $\omega_c=\omega_p$.

calculated from Eq. (24) for other relative values of ω_c and ω_p.

From Eq. (23) we may now determine the residues at the two poles, ω_1 and ω_2. They are given by the expressions

$$R_{1,2}=\frac{\omega_p^2(c^2q^2+\omega_p^2-\omega_{1,2}^2)}{2\omega_{1,2}(\omega_1^2-\omega_2^2)}. \tag{26}$$

Finally, the differential cross section (per particle) can be written in the form

$$\frac{d^2\sigma}{d\Omega d\omega}=\frac{1}{2\pi}\left(\frac{q^2E_F}{4\pi ne^2}\right)\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{h\omega_p}{E_F}\right) \times\frac{1}{\omega_p}[R_1\delta(\omega-\omega_1)+R_2\delta(\omega-\omega_2)]. \tag{27}$$

Plots of the residues R_1 and R_2 versus q for the case $\omega_c=\omega_p$ are shown in Fig. 2. It is interesting that in the limit of small q both modes have an appreciable oscillator strength. As q increases the oscillator strength of the electromagnetic-type mode decreases quite rapidly. The strength of the magnetoplasma mode, on the other hand, increases and finally levels out for large q at the value predicted from the quasistatic formula [Eq.

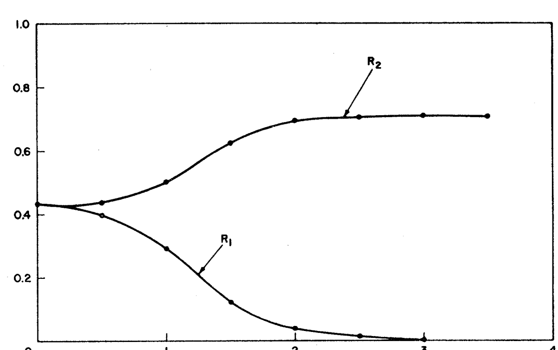

FIG. 2. Oscillator strengths of the plasma-polariton in the case $\omega_c=\omega_p$.

[13] W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (MIT Press, Cambridge, Mass., 1963).

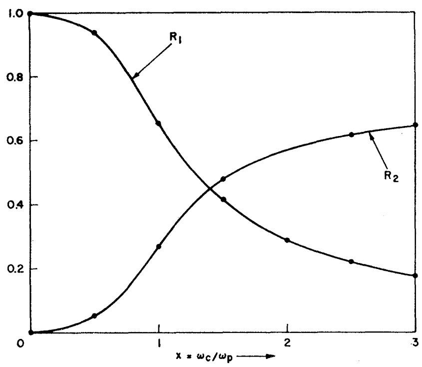

FIG. 3. Hybrid oscillator strengths for propagation at 45° to the magnetic field.

(21)]. It is important to notice that Eq. (28) contains a factor q^2. As we have emphasized, coupling of electromagnetic and plasma waves only occurs at small q, when the plasma wave is traveling at a velocity comparable to the velocity of light. On the other hand, when q is small, this screening factor greatly reduces the cross section. As an example, we may consider the case of n-type GaAs doped to 10^{18} carriers/cc. The condition $x=1$ (see Fig. 1) is achieved at a scattering angle of about 5° in such a crystal. This screening factor reduces the cross-section by about a factor of 100, compared to large-angle plasmon scattering.[3] Thus, will it be very difficult to observe the plasma-polariton.

IV. HYBRID PLASMA MODES

In several of the experiments in which light scattering from plasma waves has been observed, light was propagated at an angle to the dc magnetic field—in particular, at 45° to it. This is not an ideal geometry, but does have the interesting feature that even in the limit of fairly large q there are two types of hybrid plasma modes which might be observed. Their frequencies can easily be calculated by setting Eq. (19) equal to zero, and using the formulas of Eq. (24) to determine the components of the dielectric tensor in the local limit. The results are

$$\omega_\pm^2=\tfrac{1}{2}(\omega_c^2+\omega_p^2)\pm\tfrac{1}{2}(\omega_p^4+\omega_c^4)^{1/2}. \qquad (28)$$

These expressions have, of course, already[13] been given in the literature. It is interesting, however, to calculate the strength of the two modes in light scattering. This is a matter of evaluating Eq. (21) in this particular geometry. The strength functions normalized to those of the plasmon in the limit of zero magnetic field, are given by the formulas

$$R_\pm=\tfrac{1}{2}\omega_\pm\frac{|\omega_c^2-\omega_\pm^2|}{(\omega_p^4+\omega_c^4)^{1/2}}. \qquad (29)$$

R_+ and R_- are plotted versus magnetic field in Fig. 3. We see here, as might have been guessed in advance, that the oscillator strength shifts from one hybrid mode to another as the magnetic field increases. The crossover occurs at the field at which ω_c is about equal to ω_p. It is also clear from this graph that the lower hybrid only has appreciable oscillator strength for fields at which ω_c is greater than ω_p. This lower hybrid has recently been observed by Patel and Slusher[9] in their light-scattering experiments in n-type GaAs.

V. CONCLUSION

In this paper we have developed a general formula, which describes the scattering of light from a single-component plasma under circumstances in which the modes of the plasma can couple to electromagnetic waves. This result has been used to discuss the scattering of light from a mixed plasma-electromagnetic excitation which we call a plasma-polariton. The coupling is only strong when the plasma wave travels at a velocity comparable to the electromagnetic wave, i.e., when it has a small wave vector. Thus, one expects to observe the plasma-polariton via forward light scattering. Unfortunately, however, the whole scattering cross section of light from a plasma tends to zero as q goes to zero. This means that the plasma-polariton scattering, via electron density fluctuations, will always be exceedingly weak. In real solids, phonons are also present. These may contribute to the strength of the plasma-polariton and this contribution should eventually be estimated. However, the recent experiments of Patel and Slusher[14] suggest that, even with phonon coupling, the plasma-polariton scattering is too weak to observe.

Finally, in the last section of the paper, we have discussed the oscillator strengths for hybrid modes which propagate at 45° to the magnetic field. Both modes have been observed via light scattering. To see the lower hybrid, it is necessary to go to magnetic fields which make the cyclotron frequency comparable to a larger than the plasma frequency. This has been done in recent experiments by Patel and Slusher.

[14] C. K. N. Patel and R. E. Slusher, Phys. Rev. Letters **22**, 282 (1969).